LEVEL 6 (Calculus) BLITZ 2015

1) The slope of the tangent line to the graph of $y = x^2 \cos x$ at $\left(\frac{\pi}{2}, 0\right)$ is

a) $-\pi^2$ b) $-\frac{\pi^2}{4}$ c) 0 d) $\frac{\pi^2}{4}$ e) π^2

2) On the interval $2 < x < 3$, $y = x^3 - 6x^2 + 9x - 5$ is

a) decreasing and concave down.
b) decreasing and concave up.
c) increasing and concave down.
d) increasing and concave up.
e) horizontal.

3) $\lim_{x \to 0} \frac{\tan(3x)}{x}$ is

a) 0 b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) 3 e) does not exist

4) The concentration C of a medication h hours after it is ingested is given by $C(h) = \frac{0.3h}{(h+2.5)^2}$, where $h \geq 0$. The time when concentration will be highest is

a) 1.0 hours b) 1.5 hours c) 2.0 hours d) 2.5 hours e) 3.0 hours

5) $\int \frac{dx}{x^2+4}$ is

a) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$ b) $2\tan^{-1}\left(\frac{x}{2}\right)+C$ c) $\frac{1}{2}\tan^{-1}(2x)+C$

d) $2\tan^{-1}(2x)+C$ e) none of these

6) If $f(x) = (x^4 - 2)(x^3 - 1)(x^2 + 1)$, then $f'(-1)$ is

a) -6 b) -3 c) 0 d) 3 e) 6

7) The area of the region bounded by the curves $y = 4x - 4$ and $y = \ln x$ from $x = 1$ to $x = 2$ is

a) $3 - 2\ln 2$ b) $3 + 2\ln 2$ c) $2 - 2\ln 2$ d) $2 + 2\ln 2$ e) none of these

8) A truck moves away from the base of a 300 foot tall tower at 15 ft/sec. When the truck is 400 feet from the base of the tower, the distance between the truck and the top of the tower is increasing at

a) 10 ft/sec b) 12 ft/sec c) 15 ft/sec d) 16 ft/sec e) 20 ft/sec

9) A coefficient c for which $f(x) = \begin{cases} 5 - x, & x < 1 \\ cx - x^2, & x \geq 1 \end{cases}$ is continuous at $x = 1$ is

a) 2 b) 3 c) 4 d) 5 e) none of these

10) $\lim_{\theta \to 0} \dfrac{\cos(\cos\theta)}{\sec\theta}$ is

a) 1 b) $\sin 1$ c) $\cos 1$ d) 0 e) 2

11) A particle moves along the x - axis with a velocity given by $v(t) = 15t^2 + 8t - 2$ for $t \geq 0$. If the particle is at position $x = 6$ at $t = 0$, then the position of the particle at time $t = 1$ is

a) 7 b) 13 c) 17 d) 21 e) 27

12) $\lim_{x \to 0} \dfrac{\sqrt{1 + 8x} - 1}{x} =$

a) 2 b) $2\sqrt{2}$ c) 4 d) 8 e) does not exist

13) On the interval $[-1,1]$, the equation $x^5 - 7x + c = 0$ has

a) at most two real roots b) no real roots c) at most three real roots
d) at most one real root e) at most five real roots

14) If $3xy^2 - y + x = 7$, then y' is

a) $\dfrac{3y^2+1}{1-6xy}$ b) $\dfrac{3y^2+1}{6xy-1}$ c) $\dfrac{3y^2-1}{1-6xy}$ d) $\dfrac{3y^2-1}{6xy-1}$ e) none of these

15) Given $f(x) = \begin{cases} \cos(\pi x) & , x \le 0 \\ 1-x & , x > 0 \end{cases}$, then $\int_{-1}^{1} f(x)\,dx$ is

a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) $\frac{1}{2}+\frac{1}{\pi}$ d) $\frac{1}{2}-\frac{1}{\pi}$ e) $-\frac{1}{2}+\frac{1}{\pi}$

16) Using the substitution $u = x^2$, the integral $\int_1^{\sqrt{2}} xe^{x^2}\,dx$ is equal to

a) $\frac{1}{2}\int_1^{\sqrt{2}} e^u\,du$ b) $\frac{1}{2}\int_1^{2} e^u\,du$ c) $\int_1^{2} e^u\,du$ d) $2\int_1^{2} e^u\,du$ e) $2\int_1^{\sqrt{2}} e^u\,du$

17) $\lim\limits_{x\to -a} \dfrac{x^2 - a^2}{x^6 - a^6}$ is

a) 0 b) $-\frac{1}{a^4}$ c) $\frac{1}{a^4}$ d) $-\frac{1}{3a^4}$ e) $\frac{1}{3a^4}$

18) Let f be the function defined by $f(x) = \sqrt{|4-x|}$ for all x. The statement which is true is

a) f is not continous at $x = 4$
b) f is differentiable at $x = 4$
c) $\lim\limits_{x\to 4} f(x) \neq 0$
d) $x = 4$ is a vertical asymptote of the graph of f
e) f is continous but not differentiable at $x = 4$

19) A snowflake falls from a cloud with a velocity of $0.7t$ feet per second, where t represents time in seconds. The average velocity of the snowflake during the first 5 seconds of its fall is

a) 0.7 ft/sec b) 1 ft/sec c) 1.5 ft/sec d) 1.75 ft/sec e) 3.5 ft/sec

20) If $f(x) = \csc^{-1} x$, then $f'(x) =$

a) $\frac{x}{\sqrt{x^2-1}}$ b) $\frac{-x}{\sqrt{x^2-1}}$ c) $\frac{1}{x\sqrt{x^2-1}}$ d) $\frac{-1}{x\sqrt{x^2-1}}$ e) none of these

21) $\int_0^1 \frac{2x^3+5x}{x^4+5x^2+4}\,dx$ is

a) $\frac{1}{2}$ b) $\frac{1}{2}\ln\frac{5}{2}$ c) $\frac{3}{2}$ d) $\frac{3}{2}\ln\frac{1}{2}$ e) none of these

22) $\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}}$ is

a) 2 b) 4 c) $2\sqrt{3}$ d) $4\sqrt{2}$ e) none of these

23) The equation(s) of the horizontal asympote(s) for the curve $y = \frac{x-9}{\sqrt{4x^2+3x+2}}$ is/are

a) $y = \pm\frac{1}{4}$ b) $y = 0$ c) $y = \frac{9}{2}$ d) $y = -\frac{9}{2}$
e) none of these

24) Let $f(\theta) = \cot(\theta)$. Then $f'''\left(\frac{\pi}{6}\right)$ is

a) $4\sqrt{3}$ b) $\sqrt{3}$ c) -80 d) 20 e) none of these

25) Car A is traveling west at 50 miles per hour and Car B is traveling north at 60 miles per hour. Both cars are headed for the intersection of the two roads. The rate at which the cars are approaching each other when Car A is 0.3 miles and Car B is 0.4 miles for the intersection is

a) 39 mi/hr b) 78 mi/hr c) $10\sqrt{106}$ mi/hr d) 70 mi/hr
e) none of these

26) $\lim_{x\to 0} \dfrac{\sin 8x}{\sin 9x}$ is

a) 1 b) $\dfrac{9}{8}$ c) $\dfrac{8}{9}$ d) 0 e) none of these

27) The slope of the tangent line to the graph of $y = \dfrac{2}{1 + e^{-x}}$ at x = 0 is

a) 0 b) 1 c) 2 d) $\dfrac{1}{2}$ e) none of these

28) The average rate of change of the function $R(\theta) = \sqrt{4\theta + 1}$ on the interval $[0, 2]$ is

a) 0 b) 2 c) 3 d) 1 e) none of these

29) Let $f(x) = \begin{cases} x^2 - 1, & \text{if } -1 \le x < 0 \\ 2x, & \text{if } 0 < x < 1 \\ 1, & \text{if } x = 1 \\ -2x + 4, & \text{if } 1 < x < 2 \\ 0, & \text{if } 2 < x < 3 \end{cases}$.

Then $\lim_{x\to 1^+} f(x)$ is

a) 0 b) 1 c) 2 d) 3 e) none of these

30) $\sec(\tan^{-1}1 + \csc^{-1}1)$ is

a) 0 b) 1 c) $\frac{1}{2}$ d) $\frac{2\sqrt{3}}{3}$ e) none of these

31) Let $y = \int_{e^x}^{0} \sin^3 t \, dt$. Then $\frac{dy}{dx}$ is

a) $-\sin^3(e^x)$ b) $-e^x \sin^3(e^x)$ c) $e^x \sin^3(e^x)$ d) $\sin^3(e^x)$
e) none of these

Problems 32-34 concern the function $f(x) = x^4 - 6x^2$

32) State the interval(s) for which f is decreasing.

a) $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$ b) $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$ c) $(0, 3)$
d) $(-\infty, -3)$ e) none of these

33) State the x-value(s) of the local minimum point(s) of f.

a) 0 b) 1 c) $\pm\sqrt{3}$ d) ± 3 e) none of these

34) State the x-value(s) of the inflection point(s) of f.

a) 0 b) ± 1 c) 12 d) 10 e) none of these

35) $\lim_{x \to -\infty} \frac{\sqrt{x^2 - 9}}{2x - 6}$ is

a) $\frac{\sqrt{3}}{2}$ b) $-\frac{1}{2}$ c) $\frac{3}{2}$ d) $\frac{1}{2}$ e) none of these

36) The area of the region bound by the curves $y = x - 1$ and $y^2 = 2x + 6$ from $x = -3$ to $x = 5$ is

a) 18 b) 10 c) 5 d) 2 e) none of these

37) The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds, $t \geq 0$, and s is measured in meters. The total distance traveled by the particle during the first five seconds is

a) 20 meters b) 16 meters c) 28 meters d) 12 meters
e) none of these

38) $\int_0^1 2^{-\theta}\, d\theta$ is

a) $\frac{1}{2}$ b) $-\frac{1}{2\ln 2}$ c) $\frac{1}{\ln 2}$ d) $\frac{1}{2\ln 2}$
e) none of these

39) Let $y^2 + x^2 = y^4 - 2x$. Then the slope of the curve at the point $(-2, 1)$ is

a) -1 b) 0 c) 1 d) -2 e) none of these

40) $\lim_{h\to 0} \frac{e^{h+2} - e^2}{h}$ is

a) 0 b) 1 c) e d) e^2 e) none of these